Topic 5: Binomial Expansion

$(a+x)^0 = 1$

$(a+x)^1 = a+x$

$(a+x)^2 = a^2+2ax+x^2$

$(a+x)^3 = a^3+3a^2x+3ax^2+x^3$

$(a+x)^4 = a^4+4a^3x+6a^2x^2+4ax^3+x^4$

$(a+x)^5 = a^5+5a^4x+10a^3x^2+10a^2x^3+5ax^4+x^5$

Variables

the power = power of $(a+x)^n$

$(a+x)^0 =$

$(a+x)^1 = a \quad x$

$(a+x)^2 = a^2 \quad ax \quad x^2$

$(a+x)^3 = a^3 \quad a^2x \quad ax^2 \quad x^3$

$(a+x)^4 = a^4 \quad a^3x \quad a^2x^2 \quad ax^3 \quad x^4$

$(a+x)^5 = a^5 \quad a^4x \quad a^3x^2 \quad a^2x^3 \quad ax^4 \quad x^5$

Coefficients

$(a+x)^0 = 1$

$(a+x)^1 = 1 \quad 1$

$(a+x)^2 = 1 \quad 2 \quad 1$

$(a+x)^3 = 1 \quad 3 \quad 3 \quad 1$

$(a+x)^4 = 1 \quad 4 \quad 6 \quad 4 \quad 1$

$(a+x)^5 = 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$

$1+2=3$

Pascal's triangle

can be represented by: $\binom{n}{r} = {}^nC_r = \frac{n!}{(n-r)!\,r!}$

Some Pattern

BINOMIAL THEOREM

summation starting from $r=0$ to $r=n$

$$(a+x)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} x^r = \binom{n}{0} a^n x^0 + \binom{n}{1} a^{n-1} x^1 + \dots + \binom{n}{n} x^n$$

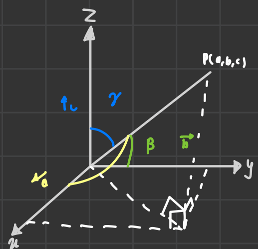
$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \dots$$

if $a=1$

$$r^{th} \text{ term}, \; T_r = \binom{n}{r-1} a^{n-(r-1)} x^{r-1}$$

Topic 8: VECTORS

Direction Cosine



$\cos\alpha = \dfrac{a}{|\vec{OP}|}$

$\cos\beta = \dfrac{b}{|\vec{OP}|}$

$\cos\gamma = \dfrac{c}{|\vec{OP}|}$

direction cosine

Scalar Product

$$a \cdot b = |a||b|\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$$

Vector/cross Product

$|a\times b| = |a||b|\sin\theta$

right hand rule,

$x \times y = z$
$y \times z = x$
$z \times x = y$
} acw

$y \times x = -z$
$z \times y = -x$
$x \times z = -y$
} ccw

Parallel vector $= |a||b|\sin 0^\circ = 0$

Perpendicular vector $= |a\times b|\sin 90^\circ = |a\times b|(1) = |a||b|$

$$a\times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

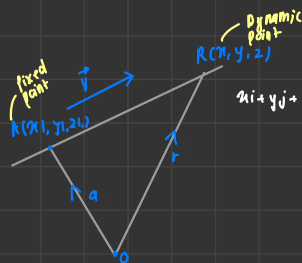
$$= (a_2b_3 - a_3b_2)i - (a_1b_3 - a_3b_1)j + (a_1b_2 - a_2b_1)k$$

Area of Parallelogram

$= |a\times b| =$

$= \dfrac{1}{2}|a\times b| =$

Equation of Lines



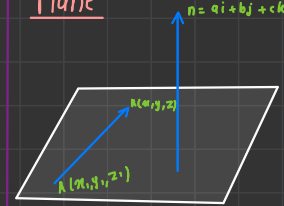
$r = a + tv$

$xi + yj + zk = (x_1i + y_1j + z_1k) + t(ai + bj + ck)$ — Vector EQN

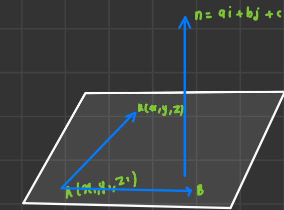
$x = x_1 + ta$
$y = y_1 + tb$
$z = z_1 + tc$
— Parametric EQN

$\dfrac{x - x_1}{a} = \dfrac{y - y_1}{b} = \dfrac{z - z_1}{c}$ — Cartesian EQN

Plane



$\vec{AR} \cdot n = 0$
$(\vec{OR} - \vec{OA}) \cdot n = 0$
$(r - a)\cdot n = 0$
$r\cdot n - a\cdot n = 0$
$r\cdot n = a\cdot n$
$r\cdot n = p$



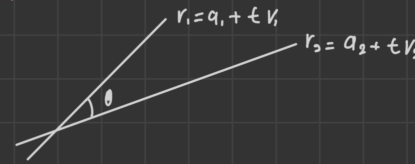
① find n,

$n = \vec{AB} \times \vec{AR}$

② find equation of plane,

$r\cdot n = p$

Angle between two straight lines



$$\theta = \cos^{-1}\frac{V_1\cdot V_2}{|V_1||V_2|}$$

Topic 9: Limits and Continuity

limit of rational function

Factorization

$$= \lim_{x \to 3} \frac{x^2 - 9}{x - 3}$$

$$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}}$$

Multiplication of Conjugate

$$= \lim_{x \to 1} \frac{x-1}{2(\sqrt{x}-1)}$$

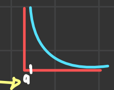
$$= \lim_{x \to 1} \frac{x-1}{2(\sqrt{x}-1)} \cdot \frac{(\sqrt{x}+1)}{(\sqrt{x}+1)}$$

$$= \lim_{x \to 1} \frac{\cancel{x-1}(\sqrt{x}+1)}{2\cancel{(x-1)}}$$

Asymptotes

Vertical

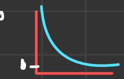
- denominator = 0
- $\lim_{x \to a^-} = \infty$



as x approaches a, y = ∞

Horizontal

- $\lim_{x \to \infty} = b$



as x approaches ∞, y = b

Continuity

i) $f(a)$ exists

ii) $\lim_{x \to a}$ exists

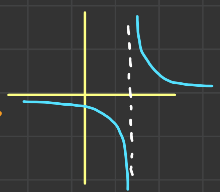
iii) $\lim_{x \to a} = f(a)$

Infinite limits

$$f(x) = \frac{1}{x-1},$$

$$\lim_{x \to 1^+} \frac{1}{x-1} = \infty$$

$$\lim_{x \to 1^-} \frac{1}{x-1} = -\infty$$



Limit at infinity

$$\lim_{x \to \infty} \frac{x^2+1}{2x^2+1} = \lim_{x \to \infty} \frac{\frac{x^2}{x^2} + \frac{1}{x^2}}{\frac{2x^2}{x^2} + \frac{1}{x^2}}$$

divide by the highest power of x, to avoid $\frac{\infty}{\infty}$

$$= \lim_{x \to \infty} \frac{1 + \frac{1}{x^2}}{2 + \frac{1}{x^2}}$$

$$= \frac{1+0}{2+0} = \frac{1}{2}$$

if $-\infty$ add (-) at eqn

$$= \lim_{x \to -\infty} \frac{x+1}{\sqrt{x^2+1}}$$

$$= \lim_{x \to -\infty} \frac{\frac{x}{x} + \frac{1}{x}}{-\sqrt{\frac{x^2}{x^2} + \frac{1}{x^2}}}$$

$$= \frac{1}{-\sqrt{1}} = -1$$

Topic 10: Differentiation

Product rule $= f(x) \cdot g(x)$

$= f(x)\,g'(x) + g(x)\,f'(x)$

quotient rule $= \dfrac{f(x)}{g(x)}$

$= \dfrac{g(x)f'(x) - f(x)\,g(x)}{(g(x))^2}$

Exponent

$y = 2e^{\sqrt{x}}$ keep e, differentiate power

$\dfrac{dy}{dx} = \cancel{2}e^{\sqrt{x}} \cdot \dfrac{1}{\cancel{2}}x^{-\frac{1}{2}}$

$= \dfrac{e^{\sqrt{x}}}{\sqrt{x}}$

Logarithmic Functions

$y = \ln(f(x))$

$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$

Special Case

$\dfrac{d}{dx}(a^{f(x)}) = a^{f(x)} \ln a \cdot f'(x)$

Trigonometry function

a) $\dfrac{d}{dx}(\sin x) = \cos x$

b) $\dfrac{d}{dx}(\cos x) = -\sin x$

c) $\dfrac{d}{dx}(\tan x) = \sec^2 x$

d) $\dfrac{d}{dx}(\cot x) = -\csc^2 x$

e) $\dfrac{d}{dx}(\sec x) = \sec x \tan x$

f) $\dfrac{d}{dx}(\csc x) = -\csc x \cot x$

Chain rule

$\dfrac{d}{dx}(\sin 3x) = \underbrace{\cos 3x}_{\frac{d}{dx}\sin 3x} \cdot \underbrace{\dfrac{d}{dx} 3x}_{\frac{d}{dx}3x}$

$= \cos 3x \cdot 3$

$= 3\cos 3x$

Power chain rule

$\dfrac{d}{dx}(\tan^3 3x) = 3\tan^2 3x \cdot \dfrac{d}{dx}\tan 3x$

$= 3\tan^2 3x \cdot \underbrace{\sec^2 3x}_{\frac{d}{dx}\tan 3x} \cdot \underbrace{3}_{\frac{d}{dx}3x}$

$= 9\tan^2 3x \sec^2 3x$

Parametric Equation

$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$

$\dfrac{d^2y}{dx^2} = \dfrac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

Stationary Point, Increasing, Decreasing Point